

Fig. 3 Amplitude spectra before and after a discontinuity (see Fig. 1).

vortex responsible for the dominant frequency; alternatively a change in character of the vortex may occur (e.g., from toroidal to helical) which can change the frequency without greatly affecting the amplitude. In either case there is a reduction in amplitude of the random intensity level.

As an alternative hypothesis to that presented in Ref. 1, in view of the similarities in the experiments, it is suggested that the reduction in noise level of Dosanjh's coaxial system might be due to a similar vortex shedding phenomenon between the inner and outer streams. At the much smaller scale of his experiment, periodic fluctuations in the flow would also be expected to be small and at a much higher frequency; such conditions would probably be difficult to see with the shadowgraph system that was used for visualization purposes in this experiment.

#### References

<sup>1</sup> Dosanjh, D. S., Yu, J. C., and Abdelhamid, A. N., "Reduction of Noise from Supersonic Jet Flows," *AIAA Journal*, Vol. 9, No. 12, Dec. 1971, pp. 2346–2353.

<sup>2</sup> Rossiter, J. E. and Kurn, A. G., "Wind Tunnel Measurements of the Effect of a Jet on the Time Average and Unsteady Pressures on the Base of a Bluff Afterbody," ARC Current Paper 903, RAE TR 65187, 1965, Royal Aircraft Establishment, Farnborough, Hampshire, England.

# Comment on "Analysis of a Clamped Skew Plate under Uniform Loading"

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THE eigenfunction expansion method employed by the authors<sup>1</sup> presents an interesting, although rather lengthy, solution to the problem of deflections of a clamped skew plate subject to uniform normal pressure.

Received May 24, 1972.

Index category: Structural Static Analysis.

It is of some interest to point out the rather little-known work of V. Ille who has investigated normally loaded skew plates by a Levy-type approach. Ille employed a simple fourth-degree polynomial that satisfied boundary conditions in the direction of two of the edges, and then obtained a relatively simple ordinary differential equation in the variable describing deflection in the direction of the other two edges. The analytical and numerical effort involved was relatively short.

For the case of the skew plate with all edges clamped, Ille<sup>2</sup> obtained a central deflection coefficient of 0.544 for a skew angle of 45°. This value lies approximately midway between the two values caused by Iyengar and Kennedy as reported in Ref. 1, and somewhat below that obtained by Kale, et al. by their eigenfunction technique. The central deflection coefficient for this same problem was found by N. L. Mikhailov<sup>3</sup> using a Bubnov-Galerkin technique to be 0.550.

It may be of some interest to note that Ille has also investigated the cases of a) uniformly loaded skew plates with two opposite edges simply supported and the other edges either simply supported or free,<sup>4</sup> and b) all four edges simply supported and with the plate having a linear thermal gradient through the thickness.<sup>5</sup> Significant stress and deflection coefficients are presented for all of these situations.

#### References

<sup>1</sup> Kale, C. S., Gopalacharyulu, S., and Rao, B. S. R., "Analysis of a Clamped Skew Plate under Uniform Loading," *AIAA Journal*, Vol. 10, No. 5, May 1972, pp. 695–697.

<sup>2</sup> Ille, V., "Contributions to the Study of a Plane, Oblique, Isotropic Plate in the Elastic Domain," Ph.D. dissertation, 1966, Inst. of Construction, Bucharest, Romania (in Romanian).

<sup>3</sup> Mikhailov, N. L., "Bending of Parallelogram-Shaped Plates," *Analysis for Strength, Stability, and Vibration*, Mashgiz Publishers, Moscow, Russia, 1955 (in Russian).

<sup>4</sup> Ille, V., "Method of Analysis of a Plane Plate Simply Supported on Two Edges and with Various Conditions of Support on the Other Edges," Publication 7, 1964, Polytechnic Inst. of Cluj, Romania (in Romanian).

<sup>5</sup> Ille, V., "Analysis of A Simply Supported Parallelogram Plate Subject to Temperature Variations," Publication 8, 1965, Polytechnic Inst. of Cluj, Romania (in Romanian).

## Optimum Stage Weight Distribution of Multistage Rockets

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#### Nomenclature

 $w_o$ , = gross initial weight of rocket

 $w_i$  = weight of ith stage

 $w_L = payload$ 

 $I_i$  = specific impulse of *i*th stage

 $c_i, n_i =$ structural weight coefficients

THE purpose of this Comment is to show that a conclusion reached by J. N. Srivastava in a Technical Comment published in the ARS Journal, February 1962, was in error. A

Received September 5, 1972.

Index categories: Launch Vehicle and Missile Trajectories; Launch Vehicle and Missile Mission Studies and Economics; Launch Vehicle and Missile Configurational Design.

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library search of ARS and AIAA publications since that time has indicated that no previous correction has been published.

In Ref. 1, J. J. Coleman developed the equations which must be satisfied for a vehicle with a fixed payload to achieve either a minimum liftoff weight at a fixed performance or maximum performance at fixed launch weight. In Ref. 2, J. N. Srivastava developed the equations for maximum payload with fixed launch weight and fixed performance. Because the resulting equations did not look the same in the two papers, Srivastava claimed that maximum performance results in a different set of optimal stage weights than the set for maximum payload.

The purpose of this Comment is to show that the results of Coleman and Srivastava are in fact identical and that there is no difference between the stated problems. The notations and assumptions are the same as in Ref. 1.

Equation (21) of Ref. 1 is

$$\frac{I_{1}(1-n_{1}c_{1}w_{1}^{n_{1}-1})}{(1-c_{1}w_{1}^{n_{1}-1})} \left[1-c_{1}w_{1}^{n_{1}-1}\left(\frac{w_{1}+w_{2}+w_{3}+w_{L}}{c_{1}w_{1}^{n_{1}}+w_{2}+w_{3}+w_{L}}\right)\right] = I_{2}\left[1-n_{2}c_{2}w_{2}^{n_{2}-1}\left(\frac{w_{2}+w_{3}+w_{L}}{c_{2}w_{2}^{n_{2}}+w_{3}+w_{L}}\right)\right] \tag{1}$$

which can be written, using Eq. (11) of Ref. 1.

$$\frac{I_{1}(1-n_{1}c_{1}w_{1}^{n_{1}-1})}{(1-c_{1}w_{1}^{n_{1}-1})} \left[1-c_{1}w_{1}^{n_{1}-1}\left(\frac{w_{o_{1}}}{w_{o_{1}}-w_{1}+c_{1}w_{1}^{n_{1}}}\right)\right] = I_{2}\left[1-n_{2}c_{2}w_{2}^{n_{2}-1}\left(\frac{w_{o_{1}}-w_{1}}{w_{o_{1}}-w_{1}+c_{2}w_{2}^{n_{2}}-w_{2}}\right)\right] \tag{2}$$

Writing the bracketed terms with common denominators and grouping  $w_{a_1}$  terms on the left-hand side gives

$$\frac{I_{1}(1-n_{1}c_{1}w_{1}^{n_{1}-1})}{(1-c_{1}w_{1}^{n_{1}-1})} \left[ \frac{w_{o_{1}}(1-c_{1}w_{1}^{n_{1}-1})-(w_{1}-c_{1}w_{1}^{n_{1}})}{w_{o_{1}}-w_{1}+c_{1}w_{1}^{n_{1}}} \right] = I_{2} \left[ \frac{(w_{o_{1}}-w_{1})+(c_{2}w_{2}^{n_{2}}-w_{2})-n_{2}c_{2}w_{2}^{n_{2}-1}(w_{o_{1}}-w_{1})}{w_{o_{1}}-w_{1}+c_{2}w_{2}^{n_{2}}-w_{2}} \right]$$
(3)

The numerator of the left-hand side can be factored into  $(w_{o_1} - w_1) \cdot (1 - c_1 w_1^{n_1 - 1})$ , so dividing both sides by  $(w_{o_1} - w_1)$  gives

$$\frac{I_{1}(1-c_{1}n_{1}w_{1}^{n_{1}-1})}{(w_{o_{1}}+c_{1}w_{1}^{n_{1}-1}-w_{1})} = I_{2}\left[\frac{(1-c_{2}n_{2}w_{2}^{n_{2}-1})}{(w_{o_{1}}-w_{1}+c_{2}w_{2}^{n_{2}-w_{2}})} - \frac{(w_{2}-c_{2}w_{2}^{n_{2}})}{(w_{o_{1}}-w_{1})(w_{o_{1}}-w_{1}+c_{2}w_{2}^{n_{2}-w_{2}})}\right]$$
(4)

which is identical to Eq. (12) of Ref. 2. A similar analysis holds for Eq. (22) of Ref. 1 and Eq. (13) of Ref. 2. Reversing the optimization index and the constraint function is therefore permitted in this problem, as Coleman stated.

#### References

<sup>1</sup> Coleman, J. J., "Optimum Stage Weight Distribution of Multistage Rockets," ARS Journal, Vol. 31, Feb. 1961, p. 259.

<sup>2</sup> Srivastava, J. N., "Optimum Stage Weight Distribution of Multistage Rockets," ARS Journal, Vol. 32, Feb. 1962, p. 296.

## **Errata**

### Errata: "Numerical Method for Hypersonic Internal Flow over Blunt Leading Edges and Two Blunt Bodies"

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[AIAA J. 10, 617-622 (1972)]

QUATION (15a) should read

 $U = U_n \{ [2\gamma + (\gamma - 1)U_n^2] / [(\gamma + 1)U_n^2] \} - W \cos \phi$ 

On page 621, line 28 should read:

a time step size  $\Delta T$  is chosen such that

 $\Delta T = \min[\Delta \lambda / [1.5a(M+1)]]$ 

Received September 1, 1972.

Index category: Supersonic and Hypersonic Flow.

### Erratum: "The Effect of Angle of Attack on Boundary-Layer Transition on Cones"

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[AIAA J. 10, 1127–1128 (1972)]

THE following figure replaces Fig. 2 of the subject Note.

INVESTIGATION	⊛c,deg	α, deg	Mαο
<ul> <li>PRESENT DATA</li> </ul>	15	0-20	7.4
■ PRESENT DATA	5	0-20	7.4
O STETSON & RUSHTON, REF I	8	0-10	5.5
DI CRISTINA, REF 2	8	4	10
♦ JULIUS, REF 3	10	10 & 20	4.95

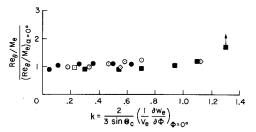


Fig. 2 Correlation of the beginning of transition on the windward ray of cones.

Received September 13, 1972.

Index category: Boundary-Layer Stability and Transition.